

# Measuring $\beta_s$ with $B_s^0 \rightarrow K^{(*)0} \bar{K}^{(*)0}$ – a Reappraisal

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## Abstract

The  $B_s^0$ - $\bar{B}_s^0$  mixing phase,  $\beta_s$ , can be extracted from  $B_s^0 \rightarrow K^{(*)0} \bar{K}^{(*)0}$ , but there is a theoretical error if the second amplitude,  $V_{ub}^* V_{us} P'_{uc}$ , is non-negligible. Ciuchini, Pierini and Silvestrini (CPS) have suggested measuring  $P_{uc}$  in  $B_d^0 \rightarrow K^{(*)0} \bar{K}^{(*)0}$ , and relating it to  $P'_{uc}$  using SU(3). For their choice of the direct and indirect CP asymmetries in  $B_d^0 \rightarrow K^{(*)0} \bar{K}^{(*)0}$ , they find that the error on  $\beta_s$  is very small, even allowing for 100% SU(3) breaking. In this paper, we re-examine the CPS method, allowing for a large range of the  $B_d^0 \rightarrow K^{(*)0} \bar{K}^{(*)0}$  observables. We find that the theoretical error in the extraction of  $\beta_s$  can be quite large, up to 18°. This problem can be ameliorated if the value of SU(3) breaking were known, and we discuss different ways, both experimental and theoretical, of determining this quantity.

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# 1 Introduction

In the standard model (SM), the weak phase of  $B_s^0$ - $\bar{B}_s^0$  mixing,  $\beta_s$ , is  $\approx 0$ . Thus, if its value is measured to be nonzero, this is a clear sign of new physics (NP). Indeed, experiments have already started measuring  $\beta_s$  in  $\bar{B}_s^0 \rightarrow J/\psi\phi$ . The results of the CDF [1] and DØ [2] collaborations hint at NP, but the errors are very large. On the other hand, the LHCb collaboration [3] finds a central value for  $\beta_s$  which is consistent with zero:  $\beta_s = (-0.03 \pm 2.89 \text{ (stat)} \pm 0.77 \text{ (syst)})^\circ$ , implying that, if NP is present in  $B_s^0$ - $\bar{B}_s^0$  mixing, its effect is small.

In the  $B_d^0$  system, the phase of  $B_d^0$ - $\bar{B}_d^0$  mixing,  $\beta$ , was first measured in the “golden mode”  $B_d^0 \rightarrow J/\psi K_S$ , and subsequently in many other modes such as  $\bar{b} \rightarrow \bar{s}$  penguin decays (e.g.  $B_d^0 \rightarrow \phi K_S$ ),  $\bar{b} \rightarrow \bar{c}c\bar{d}$  decays (e.g.  $B_d^0 \rightarrow J/\psi\pi^0$ ), etc. In the same vein, it is important to measure  $\beta_s$  in many different decay modes.

One process which is potentially a good candidate for measuring  $\beta_s$  is the pure  $\bar{b} \rightarrow \bar{s}$  penguin decay  $B_s^0 \rightarrow K^{(*)0}\bar{K}^{(*)0}$ . Its amplitude can be written

$$\mathcal{A}_s = V_{ub}^* V_{us} P'_{uc} + V_{tb}^* V_{ts} P'_{tc} . \quad (1)$$

Now, we know that  $|V_{ub}^* V_{us}|$  and  $|V_{tb}^* V_{ts}|$  are  $O(\lambda^4)$  and  $O(\lambda^2)$ , respectively, where  $\lambda = 0.23$  is the sine of the Cabibbo angle. This suggests that the  $V_{ub}^* V_{us} P'_{uc}$  term is possibly negligible compared to  $V_{tb}^* V_{ts} P'_{tc}$ . If this is justified, then there is essentially only one decay amplitude, and  $\beta_s$  can be cleanly extracted from the indirect CP asymmetry in  $B_s^0 \rightarrow K^{(*)0}\bar{K}^{(*)0}$ .

The difficulty is that it is not completely clear whether  $V_{ub}^* V_{us} P'_{uc}$  is, in fact, negligible. This term has a different weak phase than that of  $V_{tb}^* V_{ts} P'_{tc}$ , so that its inclusion will “pollute” the extraction of  $\beta_s$ . That is, if it contributes significantly to the amplitude, the value of  $\beta_s$  measured in  $B_s^0 \rightarrow K^{(*)0}\bar{K}^{(*)0}$  will deviate from the true value of  $\beta_s$ , and this theoretical error is directly related to the relative size of the two terms.

This issue has been examined by Ciuchini, Pierini and Silvestrini (CPS) in Ref. [4]. In order to get a handle on the size of  $P'_{uc}$ , CPS proceeded as follows. They considered the U-spin-conjugate decay,  $B_d^0 \rightarrow K^{(*)0}\bar{K}^{(*)0}$ , focusing specifically on  $B_d^0 \rightarrow K^{*0}\bar{K}^{*0}$ . This is a pure  $\bar{b} \rightarrow \bar{d}$  penguin decay, whose amplitude is

$$\mathcal{A}_d = V_{ub}^* V_{ud} P_{uc} + V_{tb}^* V_{td} P_{tc} . \quad (2)$$

If one takes the values for the CKM matrix elements, including the weak phases, from independent measurements, then this amplitude depends only on three unknown parameters: the magnitudes of  $P_{uc}$  and  $P_{tc}$ , and their relative strong phase. But there are three experimental measurements one can make of this decay – the branching ratio, the direct CP asymmetry, and the indirect (mixing-induced) CP asymmetry. It is therefore possible to solve for all the unknown parameters. In particular, one can obtain  $|P_{uc}|$ . This quantity can be related to  $|P'_{uc}|$  by an SU(3)-breaking factor. Now, in 2007, when Ref. [4] was written, there were no experimental measurements

of  $B_{d,s}^0 \rightarrow K^{*0} \bar{K}^{*0}$ . Instead, CPS assumed values for these measurements, inspired by QCD factorization (QCdf) [5]. They found that, even allowing for 100% SU(3) breaking, the value of  $|P'_{uc}|$  is such that the error on  $\beta_s$  due to the inclusion of a nonzero  $V_{ub}^* V_{us} P'_{uc}$  term is less than  $1^\circ$ . This inspired CPS to dub  $B_s^0 \rightarrow K^{(*)0} \bar{K}^{(*)0}$  the *golden channel* for measuring  $\beta_s$ .

In this paper, we re-examine the method of CPS. In particular, we want to establish to what extent CPS's conclusion is dependent on the values chosen for the  $B_d^0 \rightarrow K^{(*)0} \bar{K}^{(*)0}$  experimental observables. As we will see, the CPS result holds for a significant subset of the input values. However, it also fails for other choices of the inputs – the error on  $\beta_s$  due to the presence of the  $V_{ub}^* V_{us} P'_{uc}$  term can be as large as  $18^\circ$ . It is therefore not correct to say that the  $V_{ub}^* V_{us} P'_{uc}$  term has little effect, i.e. that  $\beta_s$  can always be measured cleanly in  $B_s^0 \rightarrow K^{(*)0} \bar{K}^{(*)0}$ . On the other hand, it is true that  $|P_{uc}|$  can be extracted from  $B_d^0 \rightarrow K^{(*)0} \bar{K}^{(*)0}$ . This can then be used to obtain information about  $|P'_{uc}|$  if the SU(3)-breaking factor were known reasonably accurately. We discuss different ways, both experimental and theoretical, of learning about the size of the SU(3) breaking.

In Sec. 2, we examine  $B_{d,s}^0 \rightarrow K^{(*)0} \bar{K}^{(*)0}$ , and show how the  $B_d^0$  decay can be used to obtain information about the  $B_s^0$  decay. We allow for all values of the observables in the  $B_d^0$  decay, and compute the theoretical error on  $\beta_s$ , allowing for 100% SU(3) breaking. It turns out that this error can be substantial. In Sec. 3, we discuss ways, both experimental and theoretical, of determining the SU(3) breaking. If this breaking is known with reasonable accuracy, this greatly reduces the theoretical error on  $\beta_s$ , and allows this mixing quantity to be extracted from  $B_{d,s}^0 \rightarrow K^{(*)0} \bar{K}^{(*)0}$  decays. We conclude in Sec. 4.

## 2 $B_{d,s}^0 \rightarrow K^{(*)0} \bar{K}^{(*)0}$

### 2.1 $B_s^0 \rightarrow K^{(*)0} \bar{K}^{(*)0}$

$B_s^0 \rightarrow K^{(*)0} \bar{K}^{(*)0}$  is a pure  $\bar{b} \rightarrow \bar{s}$  penguin decay. That is, its amplitude receives contributions only from gluonic and electroweak penguin (EWP) diagrams. There are three contributing amplitudes, one for each of the internal quarks  $u$ ,  $c$  and  $t$  (the EWP diagram contributes only to  $P'_t$ ):

$$\begin{aligned} \mathcal{A}_s &= \lambda_u^{(s)} P'_u + \lambda_c^{(s)} P'_c + \lambda_t^{(s)} P'_t \\ &= |\lambda_u^{(s)}| e^{i\gamma} P'_{uc} - |\lambda_t^{(s)}| P'_{tc}, \end{aligned} \quad (3)$$

where  $\lambda_q^{(q')} \equiv V_{qb}^* V_{qq'}$ . (As this is a  $\bar{b} \rightarrow \bar{s}$  transition, the diagrams are written with primes.) In the second line, we have used the unitarity of the Cabibbo-Kobayashi-Maskawa (CKM) matrix ( $\lambda_u^{(s)} + \lambda_c^{(s)} + \lambda_t^{(s)} = 0$ ) to eliminate the  $c$ -quark contribution:  $P'_{uc} \equiv P'_u - P'_c$ ,  $P'_{tc} \equiv P'_t - P'_c$ . Also, above we have explicitly written the weak-phase dependence (including the minus sign from  $V_{ts}$  in  $\lambda_t^{(s)}$ ), while  $P'_{uc}$  and  $P'_{tc}$

contain strong phases. (The phase information in the CKM matrix is conventionally parametrized in terms of the unitarity triangle, in which the interior (CP-violating) angles are known as  $\alpha$ ,  $\beta$  and  $\gamma$  [6].) The amplitude  $\bar{\mathcal{A}}_s$  describing the CP-conjugate decay  $\bar{B}_s^0 \rightarrow K^{(*)0} \bar{K}^{(*)0}$  can be obtained from the above by changing the signs of the weak phases (in this case,  $\gamma$ ).

There are three measurements which can be made of  $B_s^0 \rightarrow K^{(*)0} \bar{K}^{(*)0}$ : the branching ratio, and the direct and indirect CP-violating asymmetries. These yield the three observables

$$\begin{aligned} X' &\equiv \frac{1}{2} (|\mathcal{A}_s|^2 + |\bar{\mathcal{A}}_s|^2) , \\ Y' &\equiv \frac{1}{2} (|\mathcal{A}_s|^2 - |\bar{\mathcal{A}}_s|^2) , \\ Z'_I &\equiv \text{Im} \left( e^{-2i\beta_s} \mathcal{A}_s^* \bar{\mathcal{A}}_s \right) . \end{aligned} \quad (4)$$

Assuming one takes the values for  $|\lambda_u^{(s)}|$ ,  $|\lambda_t^{(s)}|$  and  $\gamma$  from independent measurements,  $\mathcal{A}_s$  then depends only on the magnitudes of  $P'_{uc}$  and  $P'_{tc}$ , and their relative strong phase  $\delta'$ . With  $\beta_s$ , this makes a total of four unknown parameters. These cannot be determined from only three observables – additional input is needed.

Note that, if  $\lambda_u^{(s)} P'_{uc}$  were negligible, we would only have two unknowns –  $|P'_{tc}|$  and  $\beta_s$ . These could be determined from the measurements of  $X'$  and  $Z'_I$  ( $Y'$  would vanish). This demonstrates that if one extracts  $\beta_s$  from  $Z'_I$  assuming that  $\lambda_u^{(s)} P'_{uc}$  is negligible, and it is not, then one will obtain an incorrect value for  $\beta_s$ . The size of this error is directly related to the size of  $\lambda_u^{(s)} P'_{uc}$ . Here, the possibility of an error is particularly important. Since  $\beta_s \approx 0$  in the SM, a nonzero measured value of  $\beta_s$  would indicate NP<sup>5</sup>. It is therefore crucial to have this theoretical uncertainty under control.

## 2.2 $B_d^0 \rightarrow K^{(*)0} \bar{K}^{(*)0}$

In order to deal with the  $P'_{uc}$  problem in  $B_s^0 \rightarrow K^{(*)0} \bar{K}^{(*)0}$ , in Ref. [4], CPS use its U-spin-conjugate decay  $B_d^0 \rightarrow K^{(*)0} \bar{K}^{(*)0}$ . This is a pure  $\bar{b} \rightarrow \bar{d}$  penguin decay, whose amplitude can be written

$$\mathcal{A}_d = |\lambda_u^{(d)}| e^{i\gamma} P_{uc} + |\lambda_t^{(d)}| e^{-i\beta} P_{tc} . \quad (5)$$

As with  $\mathcal{A}_s$ , we take the values for the magnitudes and weak phases of the CKM matrix elements from independent measurements. This leaves three unknown parameters in  $\mathcal{A}_d$ : the magnitudes of  $P_{uc}$  and  $P_{tc}$ , and their relative strong phase  $\delta$ . And, as with  $B_s^0 \rightarrow K^{(*)0} \bar{K}^{(*)0}$ , there are three measurements which can be made

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<sup>5</sup>Note that, if there is an indication of NP, we will know that it is in  $\bar{b} \rightarrow \bar{s}$  transitions. However, we will not know if  $B_s^0$ - $\bar{B}_s^0$  mixing and/or the  $\bar{b} \rightarrow \bar{s}$  penguin amplitude is affected.

of  $B_d^0 \rightarrow K^{(*)0} \bar{K}^{(*)0}$ : the branching ratio, and the direct and indirect CP-violating asymmetries. Given an equal number of observables and unknowns, we can solve for  $|P_{uc}|$ ,  $|P_{tc}|$  and  $\delta$ .

The key point is that  $|P_{uc}|$  and  $|P'_{uc}|$  are equal under flavor SU(3) symmetry. Thus, given a value for  $|P_{uc}|$  and a value (or range) for the SU(3)-breaking factor, one obtains the value (or range) of  $|P'_{uc}|$ . With this, one can extract the true value (or range) of  $\beta_s$  from the  $B_s^0 \rightarrow K^{(*)0} \bar{K}^{(*)0}$  experimental data.

Now, CPS focused mainly on the decays  $B_{d,s}^0 \rightarrow K^{*0} \bar{K}^{*0}$ . As mentioned in the introduction, there were no experimental measurements of these decays when their paper was written, so it was necessary to assume experimental values in order to extract  $|P_{uc}|$ . CPS chose values roughly based on the QCDf calculation of Ref. [7]. They found that the value of  $|P_{uc}|$  is such that, even allowing for 100% SU(3) breaking,  $|\lambda_u^{(s)} P'_{uc}|$  is indeed small. The upshot is that the theoretical uncertainty in the extraction of  $\beta_s$  is less than 1°.

There are several reasons not to take this result at face value. First, although QCDf has been very successful at describing the  $B$ -decay data, it is still a model. Indeed, the predictions and explanations of other models of QCD – perturbative QCD [8] (pQCD) and SCET [9], for example – do not always agree with those of QCDf. Second, QCDf assumes that factorization holds to leading order for all  $B$  decays. However,  $B_{d,s}^0 \rightarrow K^{(*)0} \bar{K}^{(*)0}$  are penguin decays, and it has been argued that non-factorizable effects are important for such decays. It may be that sub-leading effects in QCDf are, in fact, important for  $B_d^0 \rightarrow K^{*0} \bar{K}^{*0}$ . We therefore re-examine the CPS method taking a more model-independent approach.

## 2.3 Theoretical Uncertainty on $\beta_s$

In this subsection, we generalize the CPS method. First, we consider all final states in  $B_{d,s}^0 \rightarrow K^{(*)0} \bar{K}^{(*)0}$ . Second, we scan over a large range of experimental input values.

We proceed as follows. The three experimental measurements of the  $B_d^0$  decay correspond to the three observables

$$\begin{aligned} X &\equiv \frac{1}{2} (|\mathcal{A}_d|^2 + |\bar{\mathcal{A}}_d|^2) \\ &= |\lambda_u^{(d)}|^2 |P_{uc}|^2 + |\lambda_t^{(d)}|^2 |P_{tc}|^2 - 2|\lambda_u^{(d)}||\lambda_t^{(d)}||P_{uc}||P_{tc}| \cos \delta \cos \alpha , \\ Y &\equiv \frac{1}{2} (|\mathcal{A}_d|^2 - |\bar{\mathcal{A}}_d|^2) = -2|\lambda_u^{(d)}||\lambda_t^{(d)}||P_{uc}||P_{tc}| \sin \delta \sin \alpha , \\ Z_I &\equiv \text{Im} \left( e^{-2i\beta} \mathcal{A}_d^* \bar{\mathcal{A}}_d \right) = |\lambda_u^{(d)}|^2 |P_{uc}|^2 \sin 2\alpha - 2|\lambda_u^{(d)}||\lambda_t^{(d)}||P_{uc}||P_{tc}| \cos \delta \sin \alpha . \end{aligned} \tag{6}$$

It is useful to define a fourth observable:

$$\begin{aligned} Z_R &\equiv \text{Re} \left( e^{-2i\beta} \mathcal{A}_d^* \bar{\mathcal{A}}_d \right) \\ &= |\lambda_u^{(d)}|^2 |P_{uc}|^2 \cos 2\alpha + |\lambda_t^{(d)}|^2 |P_{tc}|^2 - 2|\lambda_u^{(d)}||\lambda_t^{(d)}||P_{uc}||P_{tc}| \cos \delta \cos \alpha . \end{aligned} \tag{7}$$

The quantity  $Z_R$  is not independent of the other three observables:

$$Z_R^2 = X^2 - Y^2 - Z_I^2 . \quad (8)$$

Thus, one can obtain  $Z_R$  from measurements of  $X$ ,  $Y$  and  $Z_I$ , up to a sign ambiguity.

$X$ ,  $Y$  and  $Z_I$  are related to the branching ratio ( $\mathcal{B}_d$ ), the direct CP asymmetry ( $C_d$ ) and the indirect CP asymmetry ( $S_d$ ) of  $B_d^0 \rightarrow K^{(*)0} \bar{K}^{(*)0}$  as follows:

$$X = \kappa_d \mathcal{B}_d , \quad Y = \kappa_d \mathcal{B}_d C_d , \quad Z_I = \kappa_d \mathcal{B}_d S_d , \quad (9)$$

where

$$\kappa_d = \frac{8\pi m_{B_d}^2}{\tau_d p_c} . \quad (10)$$

In the above,  $m_{B_d}$  and  $\tau_d$  are the mass and the lifetime of the decaying  $B_d^0$  meson, respectively, and  $p_c$  is the momentum of the final-state mesons in the rest frame of the  $B_d^0$ .

From Eqs. (6) and (7), the quantity  $|P_{uc}|$  can then be written in terms of the observables as

$$|P_{uc}|^2 = \frac{1}{|\lambda_u^{(d)}|^2} \frac{Z_R - X}{\cos 2\alpha - 1} = \frac{\kappa_d \mathcal{B}_d}{|\lambda_u^{(d)}|^2} \frac{\pm \sqrt{1 - C_d^2 - S_d^2} - 1}{\cos 2\alpha - 1} . \quad (11)$$

The value of  $\alpha$  is not known exactly, but we know from independent measurements that it is approximately  $90^\circ$ . In what follows, we fix  $\alpha$  to  $90^\circ$  for simplicity. Note that any deviation of  $\alpha$  from this value decreases the denominator in Eq. (11), and thus makes  $|P_{uc}|$  larger. The above expression allows us to calculate  $|P_{uc}|$  for a given set of observables.

On the whole, the decays  $B_d^0 \rightarrow K^{(*)0} \bar{K}^{(*)0}$  have not yet been measured. One exception is  $B_d^0 \rightarrow K_S K_S$ . From BaBar [10], we have

$$\mathcal{B}_d = (1.08 \pm 0.28 \pm 0.11) \times 10^{-6} , \quad S_d = -1.28_{-0.73-0.16}^{+0.80+0.11} , \quad C_d = -0.40 \pm 0.41 \pm 0.06 , \quad (12)$$

while Belle finds [11]

$$\mathcal{B}_d = (0.87_{-0.20}^{+0.25} \pm 0.09) \times 10^{-6} , \quad S_d = -0.38_{-0.77}^{+0.69} \pm 0.09 , \quad C_d = 0.38 \pm 0.38 \pm 0.05 . \quad (13)$$

We see that essentially all values of  $\sqrt{C_d^2 + S_d^2}$  are still experimentally allowed.

Here is an example of the calculation of  $|P_{uc}|$  using Eq. (11). We take the QCDf-inspired central value of CPS for the branching ratio ( $\mathcal{B}_d = 5 \times 10^{-7}$ )<sup>6</sup>, and also take

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<sup>6</sup>In fact, the branching ratio for  $B_d^0 \rightarrow K^{*0} \bar{K}^{*0}$  has been measured [12]. The world average is  $\mathcal{B}_d = (8.1 \pm 2.3) \times 10^{-7}$  [13]. In order to make the generalization of the CPS method more direct, in our analysis we use the CPS value for  $\mathcal{B}_d$  (which differs from the experimental value by only a little more than  $1\sigma$ ).

$0 \leq \sqrt{C_d^2 + S_d^2} \leq 1$ . We compute  $|\lambda_u^{(d)}|$  using values for the various quantities taken from the Particle Data Group [6]. Including the errors on these quantities, we find that  $|P_{uc}|$  can be as large as  $1460 \pm 170$  eV ( $Z_R$  positive) or  $2060 \pm 240$  ( $Z_R$  negative). For comparison,  $|P_{uc}|$  is only about 180 eV if the CP asymmetries are also fixed at the QCDf-inspired central values of CPS. Note that the discrete ambiguity with  $Z_R$  negative corresponds to the case for which  $B_d^0$  decays are dominated by  $P_{uc}$ . On the other hand, we naively expect  $P_{tc}$  to be larger. Still, even if this solution were discarded, the results of our analysis below would not be changed fundamentally. The bottom line is that  $|P_{uc}|$  can, in fact, be large in  $B_d^0 \rightarrow K^{(*)0} \bar{K}^{(*)0}$  decays.

We now return to  $B_s^0 \rightarrow K^{(*)0} \bar{K}^{(*)0}$  decays. Even if  $|P_{uc}|$  is large in  $B_d^0$  decays, because of the  $|\lambda_u^{(s)}|$  CKM suppression it is not clear whether or not  $|\lambda_u^{(s)} P'_{uc}|$  really plays a significant role in the  $B_s^0$  decays. In order to ascertain this, we proceed as follows. We apply the CPS method, but consider all possible values of the observables in both  $B_d^0$  and  $B_s^0$  decays<sup>7</sup>. Thus, we use flavor SU(3) symmetry to relate  $|P_{uc}|$  and  $|P'_{uc}|$ , allowing for a 100% symmetry breaking. In order to study the worst-case scenario (the largest possible value of  $|P'_{uc}|$  within 100% breaking), we fix  $|P'_{uc}| = 2|P_{uc}|$ . Thus, for example, for the case where the branching ratio  $\mathcal{B}_d$  is taken to be the QCDf-inspired central value of CPS, but the CP asymmetries take all possible values,  $|P'_{uc}|$  can be as large as 2920 eV ( $Z_R$  positive) or 4120 eV ( $Z_R$  negative).

Given the worst-case value of  $|P'_{uc}|$ , assuming the CKM phases to be known, and fixing  $\beta_s = 0$  (in order to study the worst-case prediction in the SM), only two parameters are left unknown in the  $B_s^0 \rightarrow K^{(*)0} \bar{K}^{(*)0}$  decay. These can be extracted from the branching ratio  $\mathcal{B}_s$  and the direct CP asymmetry  $C_s$  (up to discrete ambiguities, but this does not affect the following discussion). Once this is done, all the theoretical parameters in  $B_s^0 \rightarrow K^{(*)0} \bar{K}^{(*)0}$  are known, and we can compute the time-dependent CP asymmetry  $S_s$  and the effective phase  $\beta_s^{eff}$  ( $\arg(\bar{\mathcal{A}}_s/\mathcal{A}_s)$ ). Thus we get an evaluation of the (worst-case) theoretical uncertainty of  $\beta_s$  as extracted from the time-dependent CP asymmetry of  $B_s^0 \rightarrow K^{(*)0} \bar{K}^{(*)0}$  decays.

We now present figures showing the worst-case  $\beta_s^{eff}$  in various situations. The aim is to scan over the whole observable space in order to ascertain how large  $\beta_s^{eff}$  can be within the SM. In Fig. 1, we fix both branching ratios to the CPS central values, and give the worst-case value of  $\beta_s^{eff}$  as a function of  $|P'_{uc}|$  and the direct CP asymmetry  $C_s$ . The effective phase is roughly proportional to  $|P'_{uc}|$  and can be up to  $10^\circ$  in this restricted scenario. In Fig. 2, we repeat the calculation but also allow the branching ratios to vary, presenting  $\beta_s^{eff}$  as a function of  $C_s$  and the ratio of branching ratios ( $\mathcal{B}_s/\mathcal{B}_d$ ). In this case, for the central maximum value of  $|P_{uc}|$ , an effective phase of up to  $12^\circ$  ( $Z_R$  positive) or  $18^\circ$  ( $Z_R$  negative) is obtained. In

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<sup>7</sup>In fact, the branching ratio for  $B_s^0 \rightarrow K^{*0} \bar{K}^{*0}$  has been measured [14]. Its value is  $(2.81 \pm 0.46 \text{ (stat)} \pm 0.45 \text{ (syst)} \pm 0.34 \text{ (} f_s/f_d \text{)}) \times 10^{-5}$ . The CPS value for  $\mathcal{B}_s$ , which we use in our analysis, is  $1.18 \times 10^{-5}$ .

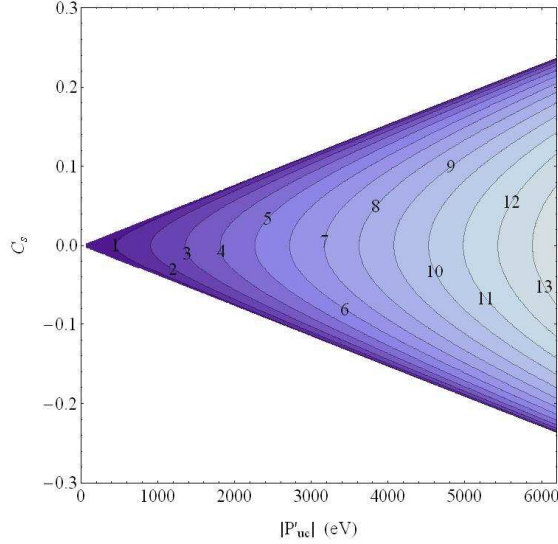


Figure 1: Worst-case values of  $\beta_s^{eff}$  (in degrees) as a function of  $|P'_{uc}|$  and the direct CP asymmetry  $C_s$ . The branching ratios are fixed to  $\mathcal{B}_d = 5 \times 10^{-7}$  and  $\mathcal{B}_s = 11.8 \times 10^{-6}$  (central values of CPS).

Fig. 3,  $\beta_s^{eff}$  is given as a function of  $\sqrt{C_d^2 + S_d^2}$  and  $\mathcal{B}_s/\mathcal{B}_d$ .

From the above figures, it is clear that  $\beta_s^{eff}$  can be large within the SM, and that the conclusions of CPS hold only for certain sets of values of the experimental inputs. Still, it is interesting to note that a small theoretical error (say  $\beta_s^{eff} \leq 5^\circ$ ) is found for a non-negligible subset of the input numbers. The general behavior of solutions is as follows:

1. for  $Z_R$  positive,  $\beta_s^{eff}$  is smaller for smaller values of  $\sqrt{C_d^2 + S_d^2}$  (it's the opposite for  $Z_R$  negative),
2.  $\beta_s^{eff}$  is smaller for larger values of  $|C_s|$  for fixed  $|P'_{uc}|$ ,
3.  $\beta_s^{eff}$  is smaller for larger values of  $\mathcal{B}_s/\mathcal{B}_d$ ,
4.  $\beta_s^{eff}$  is smaller for smaller values of SU(3) breaking.

For the first three points we cannot do anything – the measurements of the observables are what they are. The fourth point can be understood as follows. The theoretical error  $\beta_s^{eff}$  is due to the presence of a nonzero  $P'_{uc}$  in  $\mathcal{A}_s$  [Eq. (3)]. This error is roughly proportional to  $|P'_{uc}|$ , which is itself equal to the product of  $|P_{uc}|$  and an SU(3)-breaking factor. For a given value of  $|P_{uc}|$ ,  $\beta_s^{eff}$  is smaller if the SU(3)-breaking factor is smaller. Thus, the assumption of CPS of 100% breaking often leads to a large  $\beta_s^{eff}$ . The precise knowledge of the SU(3) breaking between  $|P_{uc}|$  and  $|P'_{uc}|$  would therefore considerably reduce the theoretical uncertainty on



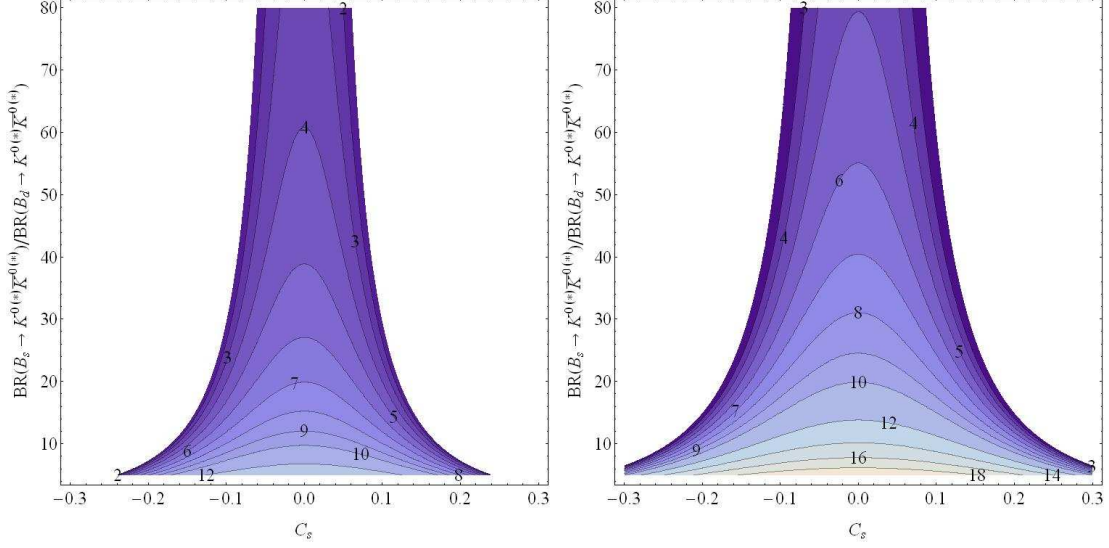


Figure 2: Worst-case values of  $\beta_s^{eff}$  (in degrees) as a function of the direct CP asymmetry  $C_s$  and the ratio of branching ratios ( $\mathcal{B}_s/\mathcal{B}_d$ ). The plot on the left (right) is for  $Z_R$  positive (negative) in Eq. (8).

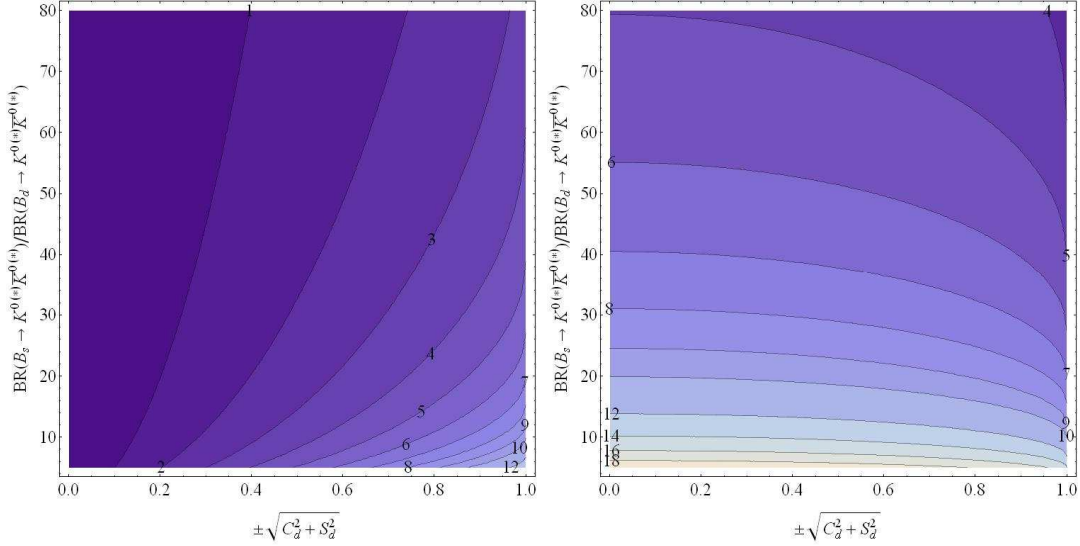


Figure 3: Worst-case values of  $\beta_s^{eff}$  (in degrees) as a function of  $\sqrt{C_d^2 + S_d^2}$  and the ratio of branching ratios ( $\mathcal{B}_s/\mathcal{B}_d$ ). The plot on the left (right) is for  $Z_R$  positive (negative) in Eq. (8).

the extracted value of  $\beta_s$  using this method. The determination of the size of SU(3) breaking is discussed in the next section.

### 3 SU(3) Breaking

As we have seen, the idea of obtaining information on  $|P'_{uc}|$  by relating it to  $|P_{uc}|$  using flavor SU(3) is tenable. However, if one simply takes an SU(3)-breaking factor of 100%, this can lead to a theoretical error on the extraction of  $\beta_s$  of up to 18°. Thus, in order to use this method, a better determination of the size of SU(3) breaking must be found. In this section, we discuss ways, both experimental and theoretical, of getting this information.

#### 3.1 Experimental Measurement of SU(3) Breaking

##### 3.1.1 $B_{d,s}^0 \rightarrow K^{*0} \bar{K}^{*0}$

The decays  $B_{d,s}^0 \rightarrow K^{(*)0} \bar{K}^{(*)0}$  really represent three types of decay – the final state can consist of  $PP$ ,  $PV$  or  $VV$  mesons ( $P$  is pseudoscalar,  $V$  is vector). Now, the CPS method applies when the final state is a CP eigenstate. For  $PP$  and  $VV$  decays, this holds. However,  $PV$  decays do not satisfy this condition. Still, these decays can be used if the  $K^{*0}/\bar{K}^{*0}$  decays neutrally. That is, we have

$$\begin{aligned} B^0 &\rightarrow \frac{1}{\sqrt{2}} (K^0 \bar{K}^{*0} + K^{*0} \bar{K}^0) \quad (\text{CP eigenstate}) \\ &\rightarrow K^0 \bar{K}^0 \pi^0 . \end{aligned} \quad (14)$$

On the other hand, the CPS method cannot be used if the  $K^{*0}/\bar{K}^{*0}$  decays to charged particles. This is because, in this case, one cannot extract  $|P_{uc}|$  from the  $B_d^0$  decay – there are more theoretical unknowns than observables.

The SM value of SU(3) breaking can be found from any single pair of decays –  $|P_{uc}|$  and  $|P'_{uc}|$  can be extracted from the  $B_d^0$  and  $B_s^0$  decays, respectively. (As we are interested in SU(3) breaking in the SM, we set  $\beta_s$  to zero.) In principle, this value of SU(3) breaking ( $|P'_{uc}|/|P_{uc}|$ ) can then be used in a different decay, and the method of the previous section can be applied. The problem here is that this approach is applicable only if the SU(3) breaking in the two decays is expected to be similar. However,  $PP$ ,  $PV$  and  $VV$  decays are all different dynamically, so that there is no a-priori reason to expect this to hold. For example, the decay of Eq. (14) is very different from the  $PP$  decay  $B^0 \rightarrow K^0 \bar{K}^0$ , and so the  $PV$  and  $PP$  SU(3) breakings are not likely to be similar.

There is one exception, and it involves the  $VV$  decays  $B_{d,s}^0 \rightarrow K^{*0} \bar{K}^{*0}$ . Since the final-state particles are vector mesons, when the spin of these particles is taken into account, these decays are in fact three separate decays, one for each polarization.

The polarizations are either longitudinal ( $A_0$ ), or transverse to their directions of motion and parallel ( $A_{\parallel}$ ) or perpendicular ( $A_{\perp}$ ) to one another. By performing an angular analysis of these decays, the three polarization pieces can be separated.

It is also possible to express the polarization amplitudes using the helicity formalism. Here, the transverse amplitudes are written as

$$\begin{aligned} A_{\parallel} &= \frac{1}{\sqrt{2}}(A_+ + A_-) , \\ A_{\perp} &= \frac{1}{\sqrt{2}}(A_+ - A_-) . \end{aligned} \quad (15)$$

However, in the SM, the helicity amplitudes obey the hierarchy [7, 15]

$$\left| \frac{A_+}{A_-} \right| = \frac{\Lambda_{QCD}}{m_b} . \quad (16)$$

That is, in the heavy-quark limit,  $A_+$  is negligible compared to  $A_-$ , so that  $A_{\parallel} = -A_{\perp}$ . Thus, one expects the SU(3) breaking for the  $\parallel$  and  $\perp$  polarizations to be approximately equal. One can therefore extract  $|P_{uc}|$  and  $|P'_{uc}|$  from the  $B_d^0$  and  $B_s^0$  decays for one of the transverse polarizations, compute the SU(3) breaking ( $|P'_{uc}|/|P_{uc}|$ ), and apply this value of SU(3) breaking to the other transverse polarization decay pair. In this way the SU(3)-breaking factor can be measured experimentally, and can be used to determine the theoretical uncertainty in the extraction of  $\beta_s$ .

### 3.1.2 $B^+ \rightarrow K^+ \bar{K}^0$ and $B^+ \rightarrow \pi^+ K^0$

Other decays which can be used to measure SU(3) breaking are the U-spin-conjugate pair  $B^+ \rightarrow K^+ \bar{K}^0$  and  $B^+ \rightarrow \pi^+ K^0$ . While it is true that these do not involve  $B_s^0$  and  $B_d^0$  mesons, both are pure penguin decays, just like  $B_{d,s}^0 \rightarrow K^{(*)0} \bar{K}^{(*)0}$ . Restricting ourselves to the  $PP$  final states, we then expect that the SU(3) breaking in  $B^+ \rightarrow K^+ \bar{K}^0$  and  $B^+ \rightarrow \pi^+ K^0$  is similar (though not necessarily equal) to that in  $B_d^0 \rightarrow K^0 \bar{K}^0$  and  $B_s^0 \rightarrow K^0 \bar{K}^0$ . The measurement of SU(3) breaking can therefore be done using the  $B^+$  decays and applied to the  $B_d^0/B_s^0$  decays.

There is a difference compared to the previous example. Since there are no indirect CP asymmetries in  $B^+$  decays, one cannot measure  $|P'_{uc}|/|P_{uc}|$ . The SU(3) breaking probed in  $B^+ \rightarrow K^+ \bar{K}^0$  and  $B^+ \rightarrow \pi^+ K^0$  is

$$\begin{aligned} -\frac{Y'}{Y} &= \frac{|\lambda_u^{(s)}| |\lambda_t^{(s)}|}{|\lambda_u^{(d)}| |\lambda_t^{(d)}|} \frac{\sin \gamma}{\sin \alpha} \frac{|P'_{uc}|}{|P_{uc}|} \frac{|P'_{tc}|}{|P_{tc}|} \frac{\sin \delta'}{\sin \delta} \\ &= \frac{|P'_{uc}|}{|P_{uc}|} \frac{|P'_{tc}|}{|P_{tc}|} \frac{\sin \delta'}{\sin \delta} . \end{aligned} \quad (17)$$

In the second line, all the CKM factors cancel due to the sine law associated with the unitarity triangle. Thus, if the  $B^+$  decay pair is used to measure the SU(3) breaking, the theoretical error in the extraction of  $\beta_s$  must be calculated relating  $|P'_{uc}||P'_{tc}|\sin\delta'$  of  $B_s^0 \rightarrow K^0\bar{K}^0$  to  $|P_{uc}||P_{tc}|\sin\delta$  of  $B_d^0 \rightarrow K^0\bar{K}^0$ .

### 3.1.3 Other SU(3) pairs

There are many other pairs of decays that are related by U spin or SU(3):  $B_d^0 \rightarrow \pi^+\pi^-$  and  $B_s^0 \rightarrow K^+K^-$ ,  $B_d^0 \rightarrow \pi^0K^0$  and  $B_s^0 \rightarrow \pi^0\bar{K}^0$ , etc. A complete list of two- and three-body decay pairs, as well as a discussion of the measurement of U-spin/SU(3) breaking, is given in Ref. [16]. For some of them we already have measurements of the breaking.

For example, consider the pair  $B_s^0 \rightarrow \pi^+K^-$  and  $B_d^0 \rightarrow \pi^-K^+$ . The measurement of the SU(3) breaking of Eq. (17) gives [16]

$$-\frac{Y'}{Y} = 0.92 \pm 0.42. \quad (18)$$

Although the error is still substantial, we see that the central value implies small SU(3) breaking. The problem is that  $B_s^0 \rightarrow \pi^+K^-$  and  $B_d^0 \rightarrow \pi^-K^+$  are not pure-penguin decays, so that it is not clear how the above SU(3) breaking is related to that in  $B_{d,s}^0 \rightarrow K^{(*)0}\bar{K}^{(*)0}$ , if at all. Still, if one measures the SU(3) breaking in several different decay pairs, it can give us a rough indication as to what to take for  $B_{d,s}^0 \rightarrow K^{(*)0}\bar{K}^{(*)0}$ .

## 3.2 Theoretical Input on SU(3) Breaking

Consider again the  $B_s^0 \rightarrow K^{(*)0}\bar{K}^{(*)0}$  amplitude, Eq. (3). If the  $t$ -quark contribution is eliminated using the unitarity of the CKM matrix, we have

$$\mathcal{A}_s = T'\lambda_u^{(s)} + P'\lambda_c^{(s)}, \quad (19)$$

where  $T' \equiv P'_u - P'_t$ ,  $P' \equiv P'_c - P'_t$ . Now, in QCDf  $T'$  and  $P'$  are calculated using a systematic expansion in  $1/m_b$ . However, a potential problem occurs because the higher-order power-suppressed hadronic effects contain some chirally-enhanced infrared (IR) divergences. In order to calculate these, one introduces an arbitrary infrared (IR) cutoff. The key observation here is that the difference  $T' - P'$  is free of these dangerous IR divergences [17]. And, although the calculation of various hadronic quantities in pQCD is different than in QCDf, the difference  $T' - P'$  is the same in both formulations. This also holds for  $T - P$  in the  $B_d^0 \rightarrow K^{(*)0}\bar{K}^{(*)0}$  amplitude.

Since  $T' - P' = P'_{uc}$  and  $T - P = P_{uc}$ , this suggests that the calculation of  $|P'_{uc}|$  and  $|P_{uc}|$  is under good control theoretically. This allows us to calculate  $|P'_{uc}|/|P_{uc}|$ ,

which gives us the theoretical prediction of SU(3) breaking. There are many quantities which enter into the calculation of  $|P'_{uc}|$  and  $|P_{uc}|$  – the renormalization scale  $\mu$ , the Gegenbauer coefficients in the light-cone distributions, the quark masses, etc. – and the errors on these quantities are quite large at present. However, most of these quantities and their errors cancel in the ratio  $|P'_{uc}|/|P_{uc}|$ . For the various  $B_{d,s}^0 \rightarrow K^{(*)0} \bar{K}^{(*)0}$  decays we find

$$\begin{aligned}
PP & : \frac{|P'_{uc}|}{|P_{uc}|} = \frac{M_{B_s}^2 F_0^{B_s^0 \rightarrow K}(M_K^2)}{M_{B_d}^2 F_0^{B_d^0 \rightarrow K}(M_K^2)} = 0.86 \pm 0.15 , \\
PV & : \frac{|P'_{uc}|}{|P_{uc}|} = \frac{M_{B_s}^2 F_+^{B_s^0 \rightarrow K}(M_{K^*}^2)}{M_{B_d}^2 F_+^{B_d^0 \rightarrow K}(M_{K^*}^2)} = 0.86 \pm 0.15 , \\
VP, VV_0 & : \frac{|P'_{uc}|}{|P_{uc}|} = \frac{M_{B_s}^2 A_0^{B_s^0 \rightarrow K^*}(M_{K^{(*)}}^2)}{M_{B_d}^2 A_0^{B_d^0 \rightarrow K^*}(M_{K^{(*)}}^2)} = 0.87 \pm 0.19 , \\
VV_{\parallel}, VV_{\perp} & : \frac{|P'_{uc}|}{|P_{uc}|} = \frac{M_{B_s}(F_-^{B_s^0 \rightarrow K^*}(M_{K^*}^2) \pm F_+^{B_s^0 \rightarrow K^*}(M_{K^*}^2))}{M_{B_d}(F_-^{B_d^0 \rightarrow K^*}(M_{K^*}^2) \pm F_+^{B_d^0 \rightarrow K^*}(M_{K^*}^2))} = 0.79 \pm 0.16 .
\end{aligned} \tag{20}$$

Above we have taken  $F^{B \rightarrow K^{(*)}}(M_{K^{(*)}}^2) \simeq F^{B \rightarrow K^{(*)}}(0)$  since the variation of the ratio of form factors over this range of  $q^2$  falls well within the errors of their calculation [7, 18]. For  $PV$  and  $VP$  decays, the spectator quark goes in the first meson. In the last expression, we have  $F_+^{B \rightarrow K^*} = 0.00 \pm 0.06$  [7], so that one has the same SU(3) breaking for the  $\parallel$  and  $\perp$  polarizations. As discussed in Sec. 3.1.1, this is to be expected.

Now, the QCDf calculation is to  $O(\alpha_s)$ , and the above expression indicates that, to this order, the SU(3)-breaking term is factorizable. Thus, the theoretical prediction is fairly robust. On the other hand, SCET says that there are long-distance contributions to  $P'_c$  and  $P_c$ . Although this could introduce some uncertainty into  $|P'_{uc}|/|P_{uc}|$ , there might also be a partial cancellation in the ratio. Our point here is that, though one generally wants to avoid theoretical input, since this is largely based on models, the SU(3) breaking in  $|P'_{uc}|/|P_{uc}|$  may be theoretically clean.

In Sec. 3.1.1, it was noted that the CPS method can be used when the final state in  $B_{d,s}^0 \rightarrow K^{(*)0} \bar{K}^{(*)0}$  is a CP eigenstate. Thus, if one wishes to use the theoretical input of Eq. (20), one can simply apply it to  $PP$  or  $VV$  decays. However, this does not hold for  $PV$  or  $VP$  decays, which are not CP eigenstates. Still, one can use the CPS method on the decay of Eq. (14), which is a linear combination of  $PV$  and  $VP$  states. And, since the theoretical  $PV$  SU(3) breaking in Eq. (20) is about equal to that of  $VP$ , this theoretical input can be applied to the  $PV + VP$  decay.

Finally, in Sec. 2.1 we noted that, in the presence of a nonzero  $P'_{uc}$ , one cannot cleanly extract  $\beta_s$  from  $B_s^0 \rightarrow K^{(*)0} \bar{K}^{(*)0}$  – one needs additional input. In Ref. [19] it was the above theoretical calculation of  $|P'_{uc}|$  which was taken as the input. We

note that the method using  $B_s^0 \rightarrow K^{(*)0} \bar{K}^{(*)0}$  and  $B_d^0 \rightarrow K^{(*)0} \bar{K}^{(*)0}$  is somewhat more precise since most of the errors in the calculation of  $|P'_{uc}|$  cancel in the SU(3)-breaking ratio of Eq. (20).

## 4 Conclusions

The pure  $\bar{b} \rightarrow \bar{s}$  penguin decay  $B_s^0 \rightarrow K^{(*)0} \bar{K}^{(*)0}$  is potentially a good candidate for measuring the  $B_s^0$ - $\bar{B}_s^0$  mixing phase,  $\beta_s$ . If its amplitude were dominated by  $V_{tb}^* V_{ts} P'_{tc}$ , the indirect CP asymmetry would simply measure  $\beta_s$ . Unfortunately, although the second contributing amplitude,  $V_{ub}^* V_{us} P'_{uc}$ , is expected to be small, it is not clear that it is completely negligible. A nonzero  $V_{ub}^* V_{us} P'_{uc}$  can change the extracted value of  $\beta_s$  from its true value, i.e. it can lead to a theoretical error. Since the measurement of  $\beta_s$  is an important step in the search for new physics, the size of this theoretical error is important.

The size of  $P'_{uc}$  has been examined by Ciuchini, Pierini and Silvestrini (CPS). They note that the amplitude  $P_{uc}$  can be extracted from the U-spin-conjugate decay,  $B_d^0 \rightarrow K^{(*)0} \bar{K}^{(*)0}$ , and can be related to  $P'_{uc}$  by SU(3). They choose values for the  $B_d^0 \rightarrow K^{(*)0} \bar{K}^{(*)0}$  experimental observables inspired by QCdf, allow for 100% SU(3) breaking, and compute  $P'_{uc}$ . They find that the theoretical error on  $\beta_s$  is very small, i.e. that the presence of the  $V_{ub}^* V_{us} P'_{uc}$  amplitude has little effect on the extraction of  $\beta_s$ .

In this paper, we revisit the CPS method. In particular, we consider most values of the  $B_d^0 \rightarrow K^{(*)0} \bar{K}^{(*)0}$  observables, still allowing for 100% SU(3) breaking. We find that, although the theoretical error remains small for a significant subset of these input values, it can be large for other values. We find that an error of up to  $18^\circ$  is possible, which makes the extraction of  $\beta_s$  from  $B_s^0 \rightarrow K^{(*)0} \bar{K}^{(*)0}$  problematic.

This issue can be resolved if we knew the value of SU(3) breaking. We therefore discuss different ways, both experimental and theoretical, of determining this quantity. From the experimental point of view, the size of SU(3) breaking can be measured using a different  $B_d^0/B_s^0$  decay pair. We show that the  $VV$  decay  $B_{d,s}^0 \rightarrow K^{*0} \bar{K}^{*0}$  or  $B^+ \rightarrow K^+ \bar{K}^0/B^+ \rightarrow \pi^+ K^0$  can be used in this regard. It is also possible to use theoretical input. Within QCdf, the SU(3)-breaking term is factorizable, and so the theoretical prediction for this quantity may be reasonably clean.

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## References

- [1] T. Aaltonen *et al.* [CDF Collaboration], Phys. Rev. Lett. **100**, 161802 (2008) [arXiv:0712.2397 [hep-ex]], arXiv:1112.1726 [hep-ex].
- [2] V. M. Abazov *et al.* [DØ Collaboration], Phys. Rev. Lett. **101**, 241801 (2008) [arXiv:0802.2255 [hep-ex]], arXiv:1109.3166 [hep-ex].
- [3] The LHCb Collaboration, CERN-LHCb-CONF-2012-002. See also <https://cdsweb.cern.ch/record/1423592?ln=en>.
- [4] M. Ciuchini, M. Pierini and L. Silvestrini, Phys. Rev. Lett. **100**, 031802 (2008) [hep-ph/0703137 [HEP-PH]].
- [5] M. Beneke, G. Buchalla, M. Neubert and C.T. Sachrajda, Phys. Rev. Lett. **83**, 1914 (1999), Nucl. Phys. B **591**, 313 (2000), Nucl. Phys. B **606**, 245 (2001).
- [6] K. Nakamura *et al.* [Particle Data Group], J. Phys. G **37**, 075021 (2010).
- [7] M. Beneke, J. Rohrer and D. Yang, Nucl. Phys. B **774**, 64 (2007) [hep-ph/0612290].
- [8] Y. Y. Keum, H. n. Li and A. I. Sanda, Phys. Lett. B **504**, 6 (2001).
- [9] C. W. Bauer, D. Pirjol, I. Z. Rothstein and I. W. Stewart, Phys. Rev. D **70**, 054015 (2004).
- [10] B. Aubert *et al.* [BABAR Collaboration], Phys. Rev. Lett. **97**, 171805 (2006) [arXiv:hep-ex/0608036].
- [11] K. Abe *et al.* [Belle Collaboration], Phys. Rev. Lett. **98**, 181804 (2007) [arXiv:hep-ex/0608049]; Y. Nakahama *et al.* [Belle Collaboration], Phys. Rev. Lett. **100**, 121601 (2008) [arXiv:0712.4234 [hep-ex]].
- [12] B. Aubert *et al.* [BABAR Collaboration], Phys. Rev. Lett. **100**, 081801 (2008) [arXiv:0708.2248 [hep-ex]]; C. C. Chiang *et al.* [Belle collaboration], Phys. Rev. D **81**, 071101 (2010) [arXiv:1001.4595 [hep-ex]].
- [13] E. Barberio *et al.* [Heavy Flavor Averaging Group], arXiv:0808.1297 [hep-ex], and online update at <http://www.slac.stanford.edu/xorg/hfag>
- [14] R. Aaij *et al.* [LHCb Collaboration], Phys. Lett. B **709**, 50 (2012) [arXiv:1111.4183 [hep-ex]].
- [15] A. L. Kagan, Phys. Lett. B **601**, 151 (2004) [arXiv:hep-ph/0405134].
- [16] M. Imbeault and D. London, Phys. Rev. D **84**, 056002 (2011) [arXiv:1106.2511 [hep-ph]].

- [17] S. Descotes-Genon, J. Matias and J. Virto, Phys. Rev. Lett. **97**, 061801 (2006) [hep-ph/0603239]. See also A. Datta, M. Imbeault, D. London and J. Matias, Phys. Rev. D **75**, 093004 (2007) [hep-ph/0611280].
- [18] G. Duplancic and B. Melic, Phys. Rev. D **78**, 054015 (2008) [arXiv:0805.4170 [hep-ph]].
- [19] S. Descotes-Genon, J. Matias and J. Virto, Phys. Rev. D **76**, 074005 (2007) [Erratum-ibid. D **84**, 039901 (2011)] [arXiv:0705.0477 [hep-ph]], Phys. Rev. D **85**, 034010 (2012) [arXiv:1111.4882 [hep-ph]].